Physik-Praktikum: BRÜ

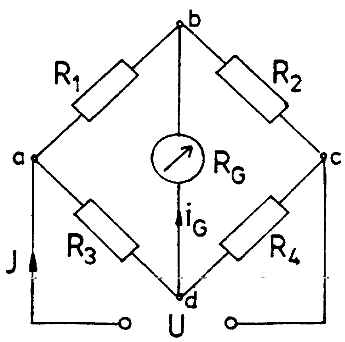
Introduction

The resistance defines the relationship between current and voltage: $U = R \cdot I$. So, to determine the resistance, only the voltage and the current have to be measured. However, because auf the internal resistances of the amps meter (that is connected in series, so the resistances add) and the voltmeter (that is connected in parallel to the resistor, so the currents add), exact measurements are impossible.
Bridge circuits are an alternative solution: if the bridge is adjusted, one resistance can be computed with the ratio of the two resistances connected in parallel and with the resistance in series; there is only an amp meter necessary to adjust the circuit, whose internal resistance does not matter because there is no current flow through when the bridge is adjusted.

With alternating current, complex numbers are used to describe non-ohmic resistors (like coils or capacitors) because there is a phase shift between current and voltage.

Experimental Set-up

In the first four experiments direct current is used with a bridge circuit:



(taken from the instructions)

The resistors R_3 and R_4 are replaced by a potentiometer with known total resistance and linear scale. The bridge is adjusted if there is no current through the amps meter.

For the measurements with alternating current (question 6) a different set-up was used: a waveform generator generates a sine waveform which is also put on the x-axis of an oscilloscope, and the amps meter is replaced by an oscilloscope (the signal is put on the y-axis of the oscilloscope). The bridge is adjusted if the figure on the screen is a horizontal line.

Results

1. Specify for the circuits shown in figure 6a and 6b the conditions for adjustment for the real and the imaginary part.

The bridge is adjusted if

$$Z_1 Z_4 = Z_2 Z_3 ;$$
$$(\mathrm{Re}\ Z_1 + j\ \mathrm{Im}\ Z_1)(\mathrm{Re}\ Z_4 + j\ \mathrm{Im}\ Z_4) = (\mathrm{Re}\ Z_2 + j\ \mathrm{Im}\ Z_2)(\mathrm{Re}\ Z_3 + j\ \mathrm{Im}\ Z_3) ;$$
$$(\mathrm{Re}\ Z_1\ \mathrm{Re}\ Z_4 - \mathrm{Im}\ Z_1\ \mathrm{Im}\ Z_4) + j\,(\mathrm{Im}\ Z_1\ \mathrm{Re}\ Z_4 + \mathrm{Im}\ Z_4\ \mathrm{Re}\ Z_1) =$$
$$= (\mathrm{Re}\ Z_2\ \mathrm{Re}\ Z_3 - \mathrm{Im}\ Z_2\ \mathrm{Im}\ Z_3) + j\,(\mathrm{Im}\ Z_2\ \mathrm{Re}\ Z_3 + \mathrm{Im}\ Z_3\ \mathrm{Re}\ Z_2) ;$$

Real part: $\mathrm{Re}\ Z_1\ \mathrm{Re}\ Z_4 - \mathrm{Im}\ Z_1\ \mathrm{Im}\ Z_4 = \mathrm{Re}\ Z_2\ \mathrm{Re}\ Z_3 - \mathrm{Im}\ Z_2\ \mathrm{Im}\ Z_3$;
Imaginary part: $\mathrm{Im}\ Z_1\ \mathrm{Re}\ Z_4 + \mathrm{Im}\ Z_4\ \mathrm{Re}\ Z_1 = \mathrm{Im}\ Z_2\ \mathrm{Re}\ Z_3 + \mathrm{Im}\ Z_3\ \mathrm{Re}\ Z_2$;

Circuit with capacitors

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\ \omega\ C_1 = \frac{1 + R_1\ j\ \omega\ C_1}{R_1}\ ;\quad Z_1 = \frac{R_1}{1 + j\ R_1\ \omega\ C_1} = \frac{R_1\,(1 - j\ \omega\ C_1)}{1 + R_1^2\ \omega^2\ C_1^2}\ ;$$

$$\text{Re}\ Z_1 = \frac{R_1}{1 + R_1^2\ \omega^2\ C_1^2}\ ;\ \text{Im}\ Z_1 = -\frac{\omega\ C_1}{1 + R_1^2\ \omega^2\ C_1^2}\ ;$$

$$Z_2 = R_2 - \frac{j}{\omega\ C_2}\ ;\ \text{Re}\ Z_2 = R_2\ ;\ \text{Im}\ Z_2 = -\frac{1}{\omega\ C_2}\ ;$$

$$Z_3 = R_3\ ;\ \text{Re}\ Z_3 = R_3\ ;\ \text{Im}\ Z_3 = 0\ ;$$

$$Z_4 = R_4\ ;\ \text{Re}\ Z_4 = R_4\ ;\ \text{Im}\ Z_4 = 0\ ;$$

So the condition for adjustment for...

– the real part is: $\frac{R_1\ R_4}{1 + R_1^2\ \omega^2\ C_1^2} = R_2\ R_3\ ;$

– the imaginary part is: $-\frac{\omega\ C_1\ R_4}{1 + R_1^2\ \omega^2\ C_1^2} = -\frac{R_3}{\omega\ C_2}\ ;$

Circuit with coils

$$\frac{1}{Z_1} = \frac{1}{R_1} - \frac{j}{\omega\ L_1} = \frac{\omega\ L_1 - j\ R_1}{R_1\ \omega\ L_1}\ ;\ Z_1 = \frac{R_1\ \omega\ L_1}{\omega\ L_1 - j\ R_1} = \frac{R_1\ \omega\ L_1\,(\omega\ L_1 + j\ R_1)}{\omega^2\ L_1^2 + R_1^2}\ ;$$

$$\text{Re}\ Z_1 = \frac{R_1\ \omega^2\ L_1^2}{\omega^2\ L_1^2 + R_1^2}\ ;\ \text{Im}\ Z_1 = \frac{R_1^2\ \omega\ L_1}{\omega^2\ L_1^2 + R_1^2}\ ;$$

$$Z_2 = R_2 + j\ \omega\ L_2\ ;\ \text{Re}\ Z_2 = R_2\ ;\ \text{Im}\ Z_2 = \omega\ L_2\ ;$$

$$Z_3 = R_3\ ;\ \text{Re}\ Z_3 = R_3\ ;\ \text{Im}\ Z_3 = 0\ ;$$

$$Z_4 = R_4\ ;\ \text{Re}\ Z_4 = R_4\ ;\ \text{Im}\ Z_4 = 0\ ;$$

So the condition for adjustment for...

– the real part is: $\frac{R_1\ R_4\ \omega^2\ L_1^2}{\omega^2\ L_1^2 + R_1^2} = R_2\ R_3\ ;$

– the imaginary part is: $\frac{R_1^2\ R_4\ \omega\ L_1}{\omega^2\ L_1^2 + R_1^2} = \omega\ L_2\ R_3\ ;$

2. Calculate the ohmic resistance of the potentiometer out of the three performed measurements. Are the values within the measunring inaccuracy?

Voltage: (0.5 ± 0.1) V

$$\frac{R_1}{R_2} = \frac{A}{1000 - A}\ ;\ R_1 = \frac{A \cdot R_2}{1000 - A}\ ;$$

comparator resistor R_2 [Ω]	***A***	***ohmic resistor R_1 [Ω]***
10	910	101
30	771	101
100	503	101

arithmetic mean value: $\bar{R}_1 = 101.11\ \Omega$
mean square deviation: $\sigma = 0.101\ \Omega$
measuring inaccuracy: $u = \sigma \cdot 0.76 = 0.0771\ \Omega$

The specified inaccuracy of the Helipot is 1% (this means 10 divisions on the scale), so the inaccuracy of the measurement is $5.9\ \Omega$. This is much less accurate than the measuring inaccuracy of our measurements.

Result: $R_1 = (101 \pm 5.9)\ \mathrm{V}$.

3. Calculate the resistance of the light bulb from the measurements with three comparator resistors. Are these values within the measuring inaccuracy?

Voltage: (3.0 ± 0.1) V

comparator resistor R_2 [Ω]	*A*	*R_1 (light bulb) [Ω]*
10	373	5.95
20	166	3.98
100	807	418
200	897	1742

measuring inaccuracy: $\frac{(A+10)\cdot R_2}{1000-(A+10)} - \frac{A \cdot R_2}{1000 - A} = \frac{907 \cdot 10}{93} - \frac{897 \cdot 10}{103} = 10.4\ \Omega$;

Obviously these values are not within the measuring inaccuracy. An explanation is given in the answer to the next question.

4. Calculate the ohmic resistance of the light bulb from the series of measurements 2c. Calculate the currents through the light bulb and plot R over I. Discuss the determined characteristic line.

$$I = \frac{U}{R_1 + R_2};$$

Voltage U [V]	*comparator resistor R_2 = 10 Ω*			*comparator resistor R_2 = 200 Ω*		
	A	*R_1 [Ω]*	*I [mA]*	*A*	*R_1 [Ω]*	*I [mA]*
0.5	792	38.1	10.4	82	17.9	2.30
1	838	51.7	16.2	86	18.8	4.57
2	878	72.0	24.4	111	25.0	8.89
3	896	86.2	31.2	161	38.4	12.6
4	908	98.7	36.8	203	50.9	15.9
5	916	109	42.0	234	61.1	19.2

The measured resistances differ very much. The diagram shows for bigger currents (more than 10 mA) a line where the resistance is proportional to the current.

The reason for this is the different temperature of the glow wire; the resistance of metal is proportional to its temperature:

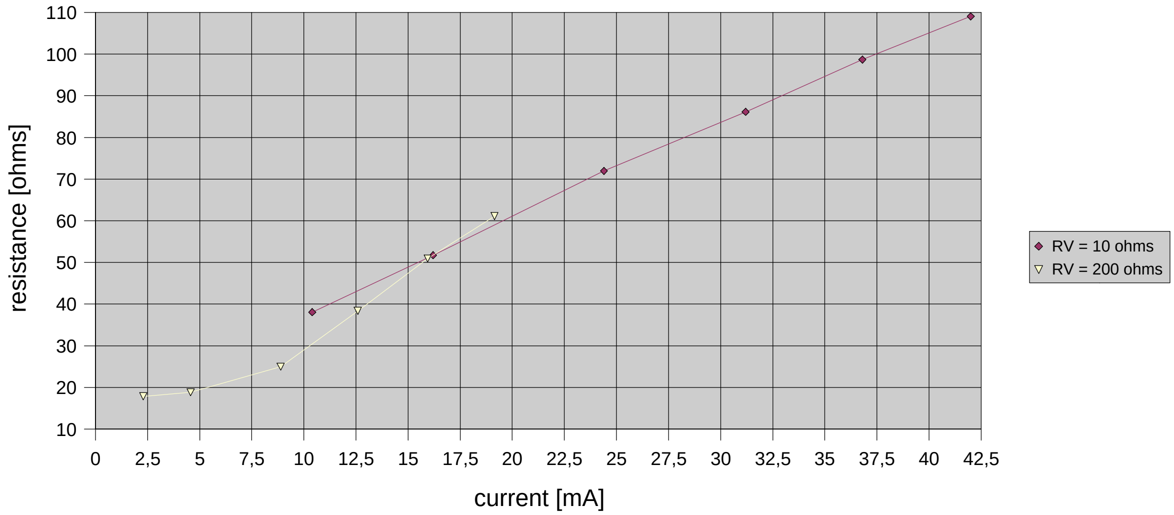
$$R(T) = R_0 + (R_0\ \beta) \cdot T\ .$$

From the diagram can be seen:

$$R_0 = 15{,}4\ \Omega\ ,\ \ R_0\ \beta = 2.25 \Rightarrow\ \beta = 0.146\ \frac{\Omega}{\mathrm{K}}\ .$$

The wire is heated by the current through it, this rises its resistance, and less current can flow – until the temperature and the resistance are balanced out. The current is determined by the resistance of the wire and the voltage.

We can only guess why this does not seem to apply for currents less than 10 mA (there the graph is not linear, and for very small currents almost horizontal). Maybe the temperature of the wire is so low that convection inside the light bulb is sufficient to cool it down.



5. How big is the ohmic resistance of the two coils?

Comparator resistor: $R_2 = 100\ \Omega$
Voltage: 0.5 V

$$R_1 = \frac{A \cdot R_2}{1000 - A}\ ;$$

	A	***R_1 [Ω]***
Small coil	783	345
Big coil	53	5.60

6. Determine the inductivities of the coils and the capacities of the capacitors. Which value do you expect theoretically for the inductivity of the half coil?

$$\frac{R_1 + j\ \omega\ L_1}{R_2 + j\ \omega\ L_2} = \frac{A}{1000 - A}\ ;$$

Small coil

$A = 660\ ;\quad R_V = 90\ \Omega\ ;\quad R_{big} = 5.60\ \Omega$ (see previous measurement); $R_2 = R_{big} + R_V\ ;$
$R_1 = R_{small} = 345\ \Omega\ ;\ L_2 = L_{big} = (0.0023 \pm 0.0001)\ \mathrm{H}\ ;$

$$R_1 + j\ \omega\ L_1 = \frac{A\left[(R_{big} + R_V) + j\ \omega\ L_2\right]}{1000 - A} = 186\ \Omega + j\ \omega \cdot 0.0045\ \mathrm{H}\ ;$$

The inductivity of the small coil is $(0.0045 \pm 0.0002)\ \mathrm{H}$.

Big coil (half coil)

$A = 112$; $R_V = 7.90\ \Omega$; $R_{small} = 345\ \Omega$; $R_2 = R_{small} + R_V$; $R_1 = R_{big} = 5.60\ \Omega$; $L_2 = 0.0045\ \text{H}$ (see previous measurement);

$$R_1 + j\ \omega\ L_1 = \frac{A\left[\left(R_{small} + R_V\right) + j\ \omega\ L_2\right]}{1000 - A} = 44.5\ \Omega + j\ \omega \cdot 0.00057\ \text{H}\ ;$$

The inductivity of half of the big coil is $(0.00057 \pm 0.00003)\ \text{H}$.

For long coils (where the length is much bigger than the diameter) one can assume:

$$L = \mu\ A\ \frac{N^2}{l}$$

(A is the cross sectional area, l the length of the coil, and N the number of turns).
So for the coil with the half number of turns we would expect approximately $1/4$ of the inductivity of the whole coil (if it has the same length; that is possible if the coil is made of several layers of wire, so using the connection in the middle half of the turns are used on the full length of the coil), that is $0.001125\ \text{H}$, if it was a long coil. But the length is about as small as the diameter, so it is not astonishing that the measured inductivitiy is only about half this value.

Capacitor

$$\frac{Z_1}{Z_2} = \frac{-\dfrac{j}{\omega\ C_1}}{-\dfrac{j}{\omega\ C_2}} = \frac{C_2}{C_1} = \frac{A}{1000 - A}\ ;\ A = 303\ ;\ C_2 = 1\ \mu\ \text{F}\ ;$$

$$\Rightarrow\ C_1 = \frac{(1000 - A)\ C_2}{A} = 2.3\ \mu\ \text{F}\ .$$