

Pohlsches Wheel and Chaos (POR)

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1 Implementation of the experiment

1.1 Determine the frequency and the constant of attenuation for the unforced linear gyration pendulum. That means the pendulum works without an additional bob and with an attenuation current of 0.4A. For that, observe the oscillation

1.1.1a) by hand

The attenuation current we used was (404 ± 1) mA. For time measurement, we used the provided mechanical stopwatch which allowed us to note down the elapsed time t after 10 oscillations of the pendulum. The precision of our measurement was $\pm 0,3$ s. This error is due to the fact that the stopwatch jumps in quite large intervalls (also around 0,3s) and the way we determined the exact moment of measurement (when the pendulum completed an oscillation and arrived at it's turning point, one of us gave a signal and the other noted the elapsed time).

measured data (10 oscillations)

#	t [s]
1	$20 \pm 0,3$
2	$20 \pm 0,3$
3	$20 \pm 0,3$
4	$20 \pm 0,3$
5	$20 \pm 0,3$

These results were always exactly 20 seconds under the conditions of the described errors. This means, one oscillation takes $(2 \pm 0,03)$ s.

Next, we measured the amplitude of the pendulum after each oscillation (10 oscillations).

measured data (amplitude)

The Amplitude A used in the following parts is bases on a circular scale which was attached around the pendulum. The scale was linear and proportional to the traveled way of the pendulum (actually the pointer). We never had measurements where it mattered on which side of the initial pendulum position the pendulum was put to an oscillation, so we will only use positive values.

	0	1	2	3	4	5	6	7	8	9	10
t [s]	0	$2 \pm 0,3$	$4 \pm 0,3$	$6 \pm 0,3$	$8 \pm 0,3$	$10 \pm 0,3$	$12 \pm 0,3$	$14 \pm 0,3$	$16 \pm 0,3$	$18 \pm 0,3$	$20 \pm 0,3$
A	$15 \pm 0,2$	$12 \pm 0,2$	$9,4 \pm 0,2$	$7,4 \pm 0,2$	$5,8 \pm 0,2$	$4,6 \pm 0,2$	$3,6 \pm 0,2$	$2,8 \pm 0,2$	$2,2 \pm 0,2$	$1,7 \pm 0,2$	$1,3 \pm 0,2$

Because we were only two students doing the experiment, we had troubles simultaneously determining the turning point of the pendulum, reading the amplitude A , the elapsed time t and writing these values down. Therefore, we assumed a time of 2 seconds per oscillation, which we measured before. Furthermore, the error in time measurement of one oscillation (due to the inaccurate stopwatch) confirmed our suspicion towards the usefulness of the measurement.

Now, only measuring the amplitude, it was even quite difficult to write down the values. So we used a trick and marked the amplitude of the pendulum per oscillation on the scale with a pencil and wrote down the values afterwards.

1.1.1.1 Frequency

The frequency of the pendulum can be calculated with the values from 1.1.1. Because we have a harmonic oscillation, the frequency is independent of the amplitude.

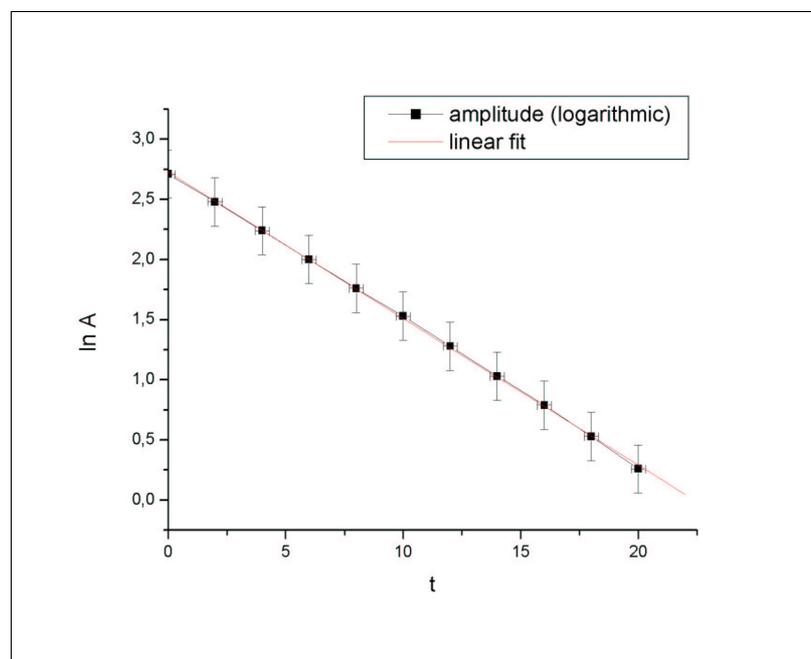
$$t_{10} = (20 \pm 0,3) \text{ s}$$

$$T_h = (2 \pm 0,03) \text{ s}$$

$$\omega_h = \frac{2 \pi}{T_h} = \frac{2 \pi}{(2 \pm 0,03) \text{ s}} = (3,1 \pm 0,05) \text{ s}^{-1}$$

1.1.1.2 Attenuation constant

The attenuation constant was computed with the mathematics software Microcal Origin 6.0. As input for the program, we used our measured data.



The equation of the calculated linear fit is:

$$y = (2,73 \pm 0,009) - (0,12 \pm 0) \cdot x$$

This gives us a value of $(-0,12 \pm 0) \text{ s}^{-1}$ for the attenuation constant λ_h . The abation constant τ_h computes to

$$\tau_h = \frac{1}{\lambda_h} = \frac{1}{(-0,12 \pm 0) \text{ s}^{-1}} = 8,33 \text{ s}$$

1.1.2b) with help of the computer

The measurement with the computer was exactly done as described in the experimental specification. We didn't change any parameters in the equipment, which means the attenuation current stayed $(404 \pm 1) \text{ mA}$. We initiated the oscillation and started the measurement after providing the required data (we set the measurement time to 22s) for the program „mess“. Except for a few mysterious „floating point“ and „abnormal program termination“ errors, we had no problem plotting the graphs.

graph01 – „classic“ oscillation

graph02 – only the oscillation peaks
graph03 – „phase space“

1.1.2.1 Frequency

We determined the period per oscillation out of „graph01“ by measuring the distance between two oscillation peaks and got the following result (the error is estimated to 0 when only two relevant digits are used):

$$T_c = (1,9 \pm 0) \text{ s}$$

$$\omega_c = \frac{2 \pi}{T_c} = \frac{2 \pi}{(1,9 \pm 0) \text{ s}} = (3,3 \pm 0) \text{ s}^{-1}$$

1.1.2.2 Attenuation constant

Also the attenuation constant was determined (indirectly) out of „graph1“. How we got the final value for τ_c can be seen in the graph. Given τ_c , the attenuation constant λ_c can be calculated:

$$\tau_c = 8,5 \text{ s}$$

$$\lambda_c = \frac{1}{\tau_c} = \frac{1}{8,5 \text{ s}} = 0,12 \text{ s}^{-1}$$

Regarding only two digits, λ_c is identical to the λ_h we calculated in 1.1.1.2.

1.1.2.3 Comparison to results from 1.1.1

Comparing the computer measured values and the hand measured values of the frequency, we get the following aberration:

$$\frac{\omega_h}{\omega_c} = (0,95 \pm 0,01)$$

...which means we have an aberration of the two values of $(5 \pm 1)\%$.

1.2 Measure the resonance graph of the pendulum with the computer and plot it. Determine the frequency of resonance by using the graph.

1.2.1 Frequency

Using the same procedure as in 1.1, we collected data with the program „mess“. We used 10 different settings of the motor speed.

motor speed	t [s]	T [s]	max	ω [s ⁻¹]
3	83 ± 0,3	8,3 ± 0,03	13	0,76 ± 0,03
4	46 ± 0,3	4,6 ± 0,03	17	1,37 ± 0,10
5	32 ± 0,3	3,2 ± 0,03	21	1,96 ± 0,20
6	25 ± 0,3	2,5 ± 0,03	31	2,51 ± 0,34
7	20 ± 0,3	2,0 ± 0,03	153	3,14 ± 0,55
8	16 ± 0,3	1,6 ± 0,03	23	3,93 ± 0,91
9	15 ± 0,3	1,5 ± 0,03	17	4,19 ± 1,05
6,5	22 ± 0,3	2,2 ± 0,03	83	2,86 ± 0,45
7,5	18 ± 0,3	1,8 ± 0,03	47	3,49 ± 0,70
7,3	19 ± 0,3	1,9 ± 0,03	85	3,31 ± 0,62

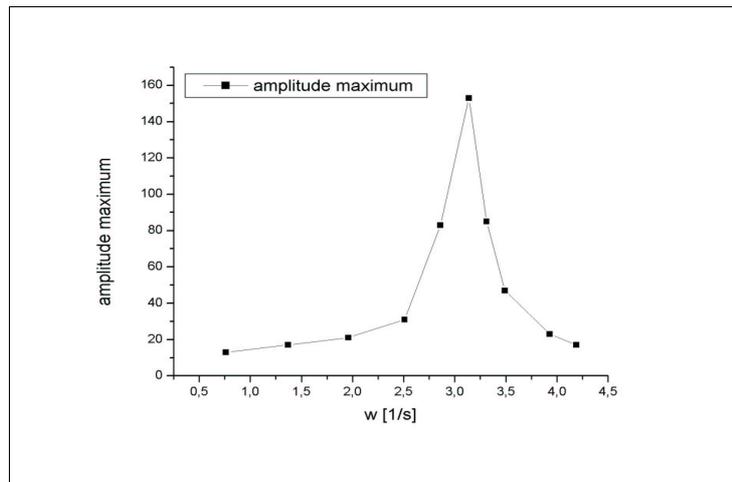
„motor speed“ are the values of the very rough und not very fine adjustable scale of the turning knob for the speed of the motor. The time t measured for 10 oscillations has the same error as in 1.1 because we used the same stopwatch. T is the period of the oscillation. „max“ are the values which were returned by the program

„mess“. They were stored in files with the extension „.max“. The frequency f was calculated out of T .

Before we started to store data, we initiated the oscillation and waited until we couldn't detect a change in the amplitude of the oscillation with our eyes (for that, we looked at the scale attached to the pendulum instrument). This way, the data which were stored in the „.max“ file should only contain a sequence of one value, our maximum. Unfortunately, this wasn't true. We had several other values in the data set, some around our maximum and some quite far away. We don't know why this is the case and therefore we searched for the „maximum“ which had the most sequential occurrences in the data set and took this value as the maximum of the oscillation.

Using those values, the following graph was plotted (as x-axis we used the frequency ω):

The graph (on paper „graph4“) shows clearly that resonance occurs at a frequency of about $\omega_r = (3,1 \pm$



$0,55) \text{ s}^{-1}$.

This is not very surprising, because it is the natural frequency of the used pendulum, as we determined it in 1.1.

1.2.2 Question: How does the deflection look like at a frequency of $\omega = 0$?

If there is no force by the motor (frequency = 0) and the pendulum is not deflected by hand then there will never be any deflection.

1.3 Verify equation 8 with the measured values.

Equation (8) in the experimental specification: $\Delta \omega = 1$

(The given equation in our specification is wrong, so we changed it to $\Delta \omega = \lambda$)

After calculating $A^* = \sqrt{2} \frac{A_{\text{max}}}{2}$ and drawing the result in the graph („graph4“), we see that $\Delta \omega = 0,15$

s^{-1} . Compared to $|\lambda| = (0,12 \pm 0) \text{ s}^{-1}$, this value is quite good for a graphical evaluation. The aberration could even be smaller, if the graph would have been drawn in a higher resolution (more measured values).

To sum up, equation (8) can be regarded as verified ;)

1.4 Mount the bob on the small aluminium pointer.

We had no problem doing this... ;)

1.5 Record the oscillation of the pendulum (which is not linear now) with the computer and compare it to the linear oscillation.

Except for the additional weight, nothing was changed in the experimental settings. Comparing the graphs, it can be seen that with additional weight, then pendulum does less oscillations in the fixed time of 30 seconds. However, it is not possible to detect a significant change in the frequency which should be related to the

amplitude because the pendulum is not linear now. A possible explanation we can think of is that a change in frequency can only be detected when the pendulum changes the side – which wasn't the case in our experiment. Therefore, we would have had to accelerate the pendulum when initiating the oscillation. Unfortunately we only let go of the pendulum at one side (at scale position 3) and recorded its subdued oscillation.

1.6 Change the attenuation current and repeat the recording of the oscillation.

First, we doubled the attenuation current to (800 ± 1) mA and printed „graph6a“ and „graph6b“. It is evident that using a higher attenuation, less oscillations are performed by the pendulum. Then we took half of the initial (404 ± 1) mA and set the attenuation current to (203 ± 1) mA. As expected, the oscillation frequency got higher again (see „graph7“).

1.7 „Play“ with the motor of the pendulum and try to find the first and second bifurcation. Record the oscillation you find interesting and plot it.

We used the attenuation current of (203 ± 1) mA and started with a very low motor speed. In the beginning, the system didn't get enough energy and the pendulum couldn't change the side. So we slowly increased the motor speed and watched how the pendulum reacted. At a certain motor speed setting (around 5 on the scale) the pendulum started to change the side and the oscillations got quite interesting (we found the first bifurcation). We tried to print this behaviour, but unfortunately the program „zeichne“ didn't print out the „typical“ phase space – only some incoherent lines were shown on the output. Bad luck for us, because these phase spaces make up really nice pictures when visualized. So we used the less interesting visualisation and printed graph 8.